Accelerated expansion in bosonic and fermionic 2D cosmologies with quantum effects

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Abstract. –

In this work we analyze the effects produced by bosonic and fermionic constituents, including quantum corrections, in two-dimensional (2D) cosmological models. We focus on a gravitational theory related to the Callan-Giddings-Harvey-Strominger model, to simulate the dynamics of a young, spatially-lineal, universe. The cosmic substratum is formed by an *inflaton* field plus a matter component, sources of the 2D gravitational field; the degrees of freedom also include the presence of a dilaton field. We show that this combination permits, among other scenarios, the simulation of a period of inflation, that would be followed by a (bosonic/fermionic) matter dominated era. We also analyse how quantum effects contribute to the destiny of the expansion, given the fact that in 2D we have a consistent (renormalizable) quantum theory of gravity. The dynamical behavior of the system follows from the solution of the gravitational field equations, the (Klein-Gordon and Dirac) equations for the sources and the dilaton field equation. Consistent (accelerated) regimes are present among the solutions of the 2D equations; the results depend strongly on the initial conditions used for the dilaton field. In the particular case where fermions are included as matter fields a transition to a decelerated expansion is possible, something that does not happen in the exclusively bosonic case.

Introduction. – Cosmological models in lower dimensions have been under analysis in several works [1–4]. These theories offer interesting mathematical results that, if properly taken into account, can also be used in realistic models. These cosmological formulations are obtained starting with different 2D gravitational models [2, 5, 6]. As a first example, we can take the gravity model proposed by Teitelboim and Jackiw [5, 9, 10] (JT model); a theory that provided consistent results at classical and quantum levels [5]. This model works as the "closest counterpart" of general relativity in the 2D case; the obvious candidate, Einsteinian gravity, furnishes no physics in 2D [1, 4, 5]. As another alternative, one can consider the CGHS model [11]; here we have an additional degree of freedom, the dilaton, giving the model the status of 2D analogue of the Brans-Dicke theory [4, 6]. Here the cosmological results include, as in the JT case, the description of a matter/radiation filled Universe [1]. The possibility of description of inflation or dark energy regimes depends basically on special conditions (such as negative energy densities) or on the use of the van der Waals equation, modeling the inflaton-matter substratum [1, 4]. Another investigations using these models considered the inclusion of quantum effects [12]; based on the remarkable fact that in 2D gravity is a renormalizable theory [5]. The one-loop corrections (integrating over scalar fields) were calculated [5, 12], and the final effective action encapsulates those effects permitting semi-classical 2D cosmological regimes [12]. In an analogous manner it is possible to consider fermions as sources; the Dirac field equations are constructed using the tetrad formalism [7] in combination with the general covariance principle. In this work, we want to analyze the positive-accelerated solutions of CGHS-inspired, 2D cosmologies with quantum corrections (without including a cosmological constant); these can be associated to the description of an inflationary period followed by the beginning of a (bosonic, fermionic) matter

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dominated period, with a compatible behavior of the physical quantities (scale factor and energy densities). We also obtain an accelerated regime when quantum corrections are considered. In fact, the behavior of the acceleration function depends strongly on the initial conditions of the dilaton field, independently of the presence of the quantum corrections. Another important point verified here is that it is possible to obtain naturally a transition to a decelerated (matter dominated) period when *fermions* are included as sources.

The manuscript is structured as follows: in Section II we make a brief review of the dilatonic 2D model with their cosmological applications. In Section III, the young 2D universe is described by taking into account the quantum corrections of the field theory, when we have bosons as sources. In the last section fermions are included, playing the role of matter constituents; this is followed by our conclusions. Units have been chosen so that $G = c = h = 1$.

II - Cosmology in 2D. – In this section we make a short review of 2D gravity in a cosmological context focusing on the CGHS model mentioned in the introduction; for further details on 2D gravity and cosmology the authors refer to results presented in [1, 4, 6]. 2D gravity has the remarkable feature that when the Einstein field equations are invoked to rule the 2D space-time physics no dynamics emerges [5]. Several theories were proposed as alternatives, based on gauge principle grounds; among them the Jackiw-Teitelboim (JT) model and the Callan-Giddings-Harvey-Strominger model (CGHS) [5]. The CGHS model was proposed initially for the investigation of 2D black-holes [11]. The action in these models include a dilaton field, which plays a role that is similar to the one present in the 4D Jordan-Brans-Dicke model [6]. The corresponding action was inspired in string theories [11], and the equations of motion that emerge are

$$G_{\mu\nu} = e^{-2\phi}\left[R_{\mu\nu} - \Lambda g_{\mu\nu} - 2\nabla_\mu\nabla_\nu\phi\right] = -8\pi T_{\mu\nu}, \;\; R - \Lambda - 4(\nabla\phi)^2 + 4\nabla^2\phi = 0, \tag{1}$$

where ϕ is the dilaton, Λ is a cosmological constant and $T_{\mu\nu}$ is the energy-momentum tensor of the sources. The Robertson-Walker metric has a simple form in a 2D Riemannian space-time $ds^2 = -(dt)^2 + a(t)^2(dx)^2$, where $a(t)$ is the cosmic scale factor, that encloses the properties of the 2D gravitational field. In fact, the usual geometrical quantities (Ricci tensor and curvature scalar) turn out to be for this metric: $R_{00} = \frac{\ddot{a}}{a}$, $R_{11} = -\ddot{a}a$, $R = 2\frac{\ddot{a}}{a}$. The regimes coming out include dust and radiation filled 2D Universes [1,6]. The existence of positive accelerated solutions depend, when barotropic equations of state are used, on the imposition of unusual conditions like negative energy [1,4]. However, interesting results appear when the van der Waals (vdW) equation is considered [4]; in fact a transition from an inflationary to a matter dominated 2D universe can be obtained in this case, as far as the wdW equation approaches the behavior of a barotropic expression as a consequence of the accelerated expansion [4]. On the other hand, the time evolution of the cosmic scale factor and of its acceleration can show, in some cases, a dramatically different behavior when compared to the one obtained in the JT model (in its gauge fixed formulation) [1, 4], as a consequence of the dilaton field behavior in this model [4]. In all cases, after the inflaton-matter field transition occurs, the accelerated regime never returns [4]. A fundamental point here is that these results arise from classical formulations, and including only bosonic sources [1, 4]. It is interesting to consider quantum effects in these models, given the fundamental point that in 2D gravity is renormalizable [5]. In the following sections we consider a model inspired in the CGHS model, including bosonic and fermionic sources with quantum effects, investigating the regimes that emerge from the 2D cosmology dynamics.

III - Bosonic 2D cosmology and Quantum effects. – Here we analyze the behaviour of a 2D cosmological theory where the gravitational sources are bosons, also investigating the influence of quantum effects in this CGHS based model (we dont include the bulk cosmological constant present in the original theory [11, 12]). The functional integration over one of the sources (the scalar field f, that will play the role of the inflaton) furnishes the one-loop corrected action, that contains the usual conformal-anomaly term plus local contributions that are included to maintain the simple form of the conserved currents [12]. The corresponding total action is written as $S = S_{\rm cl} + S_{\rm qt}$, with

$$S_{\rm cl} = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left\{e^{-2\phi}\left[R + 4(\nabla\phi)^2\right] - \frac{1}{2}\theta^2\chi^2 f^2 - \frac{1}{2}(\nabla f)^2 - \frac{1}{2}(\nabla\chi)^2\right\}, \tag{2}$$

$$S_{\rm qt} = \frac{\kappa}{2\pi}\int d^2x\sqrt{-g}\left\{-\frac{1}{4}R\frac{1}{\nabla^2}R - \frac{\gamma}{2}\phi R + q(\nabla\phi)^2\right\}, \tag{3}$$

where ϕ is the dilaton and κ is the central charge related to the conformal anomaly [5]; γ and q are constants that are

usually chosen to obtain exact solvability [12]. In action (4) we have, besides the bosonic field f, a matter constituent (the scalar field χ) whose mass will be proportional to the parameter θ.

After localization of the quantum correction (using an auxiliary field ψ [5, 12]), and plain use of the Hamilton principle, one obtains the gravitational equations of motion ($G_{\mu\nu} = T_{\mu\nu}$), which read explicitly

$$2e^{-2\phi}\left[\dot\phi^2 - \frac{\dot a}{a}\dot\phi\right] = \frac{1}{4}\dot f^2 + \frac{1}{4}\dot\chi^2 - \frac{\kappa}{2}\left(\frac{\dot a}{a}\right)^2 + \frac{\gamma\kappa}{2}\frac{\dot a}{a}\dot\phi - \frac{1}{2}q\kappa\dot\phi^2 + \frac{1}{4}\theta^2\chi^2 f^2, \tag{4}$$

$$2e^{-2\phi}\left[\ddot\phi - \dot\phi^2\right] = \frac{1}{4}\dot f^2 + \frac{1}{4}\dot\chi^2 + \kappa\left[\frac{\ddot a}{a} - \frac{1}{2}\left(\frac{\dot a}{a}\right)^2\right] - \frac{\gamma\kappa}{2}\ddot\phi - \frac{1}{2}q\kappa\dot\phi^2 - \frac{1}{4}\theta^2\chi^2 f^2. \tag{5}$$

Analogously, we obtain the equations of motion for the dilaton ϕ and scalar fields (f and χ); these are given by

$$e^{-2\phi}\left(\frac{2\ddot a}{a} + 4\dot\phi^2 - 4\ddot\phi - 4\frac{\dot a}{a}\dot\phi\right) = -\frac{\gamma\kappa}{2}\frac{\ddot a}{a} + q\kappa\left(\ddot\phi + \frac{\dot a}{a}\dot\phi\right), \tag{6}$$

$$\ddot f + \frac{\dot a}{a}\dot f + \theta^2\chi^2 f = 0, \quad \ddot\chi + \frac{\dot a}{a}\dot\chi + \theta^2\chi f^2 = 0. \tag{7}$$

Here f is playing the role of the inflaton; as mentioned before χ is the matter component, and the term associated with θ represents the direct energy transfer between both fields.

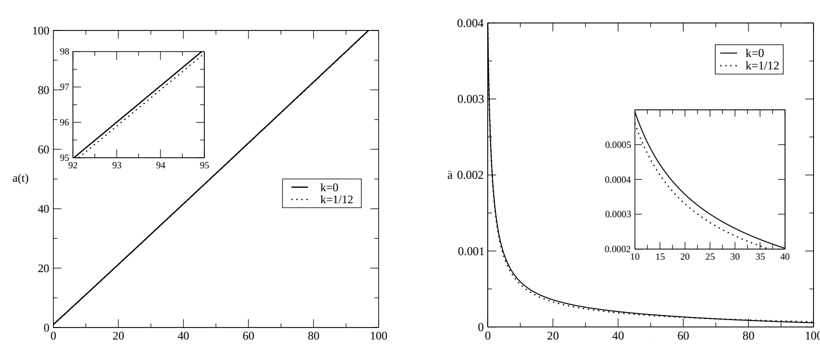
Equations (6), (7) and (8) are not linearly independent so we use the following combination of them

$$\ddot\phi + \frac{\dot a}{a}\dot\phi = 2\dot\phi^2 - \frac{1}{4}e^{2\phi}\theta^2\chi^2 f^2 - \frac{\kappa}{4}e^{2\phi}\left[q\left(\frac{\dot a}{a}\dot\phi + \ddot\phi\right) - \frac{\gamma}{2}\frac{\ddot a}{a}\right], \tag{8}$$

$$\dot\phi^2 - \frac{\ddot a}{2a} = \frac{\kappa}{4}e^{2\phi}\left[(\gamma - q)\left(\frac{\dot a}{a}\dot\phi + \ddot\phi\right) + (\gamma - 4)\frac{\ddot a}{2a}\right] + \frac{1}{4}e^{2\phi}\theta^2\chi^2 f^2. \tag{9}$$

The classical version of this model emerges by simply considering $\kappa \equiv 0$. As the expressions above show the 2D universe dynamics is ruled by a highly non-linear, coupled system (6) – (9), having exact solutions only in special cases [12]. We proceed with a numerical integration of a this system of differential equations in order to calculate the energy densities of the inflaton ρ_f and matter ρ_χ; we also obtain, accordingly, the evolution of the cosmic scale factor a, of its acceleration, and of the dilaton ϕ. The inflaton and matter energy densities are defined by

$$\rho_f = \frac{1}{4}\dot f^2 - \frac{\kappa}{2}\left[\left(\frac{\dot a}{a}\right)^2 - \gamma\dot\phi\frac{\dot a}{a} + q\dot\phi^2\right], \quad \rho_\chi = \frac{1}{4}\dot\chi^2 + \frac{1}{4}\theta^2\chi^2 f^2\,. \tag{10}$$



(a) Scale factor and

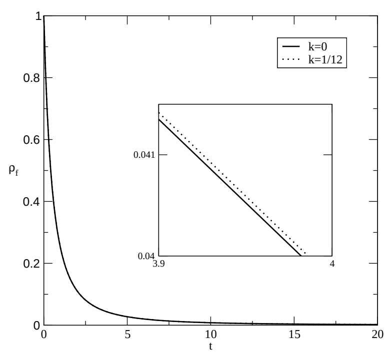
(b) acceleration vs time comparing classical with quantum corrected solutions

The boundary conditions were chosen in such a manner that the universe is qualitatively seen as initially dominated by an inflaton field with initial energy density $\rho_f(0) = 1$. The initial energy density of the matter field (χ) was considered to be zero so that the matter is created during the evolution of the universe at expenses of the inflaton

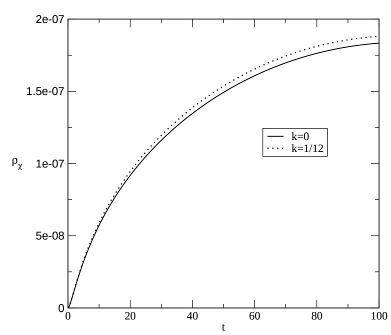
and gravitational fields. The values adopted for the initial conditions were $a(0) = 1, \dot{a}(0) = 1, f(0) = 0, \dot{f}(0) = 2.03,$ $, \chi(0) = 0.1, \dot{\chi}(0) = 0, \phi(0) = 0, \dot{\phi}(0) = 0.05$. Moreover, the values chosen for the free parameters were: (a) quantum correction-terms $\kappa = 1/12$, $q = 2$ and $\gamma = 6$ [12]; (b) coupling constant between the inflaton and matter fields $\theta = 10^{-3}$.

We start by describing the behavior of the scale factor $a(t)$. In a first situation we consider the classical model, that is, we take $\kappa = 0$. In this case the time evolution of the scale factor depends strongly on the initial value of the time derivative of the dilaton field ϕ (as had already been verified in [4]). As those values increase the expansion of the 2D universe becomes faster. On the other hand, when we turn on the quantum effects they make the expansion slower, although with the same qualitative behavior present in the classical case (see figure a). Changes in the initial value of the dilaton do not produce sensible modifications in the scale factor evolution.

Next, we analyze the behavior of the inflaton energy density ρ_f (figure c) and the matter field energy density ρ_χ (figure d). The first verification is that increasing values of $\dot{\phi}(0)$ promote a faster decrease of ρ_f independently of the presence of the quantum terms in the equations of motion; this is in tune with the behavior of the the scale factor, confirming the expansion of the 2D universe, as far as we are working with densities. For fixed values of the initial conditions it is verified that the quantum effects furnish an additional contribution to the energy density values, although without changing the qualitative character of the evolution (see figure b).



(c) Inflaton energy density and



(d) matter energy density vs time comparing classical with quantum corrected solutions

A similar effect is confirmed for the ρ_χ evolution, but the fundamental point is that this matter density is increasing with time; the transfer of energy from the inflaton and gravitational field contributions. During its evolution ρ_χ overcomes the inflaton density after a finite period of time, but unlike what happens in the 4D Einstein case this feature does not imply in a transition to a decelerated regime as we discuss bellow (see fig b).

The behavior of the acceleration $\ddot{a}$ depends strongly on the initial condition for derivative of the dilaton ϕ, as was verified in [4]. In fact, in the classical case, increasing values of $\dot{\phi}(0)$ imply into a smoother decay of $\ddot{a}$. When we consider quantum effects there is a more drastic fall in the values of $\ddot{a}$, but, as a fundamental point, only positive values are obtained, even when one is taking a long range of possibilities for the initial conditions. Besides, we also have that increasing values of $\dot{\phi}(0)$ promote a faster fall of the acceleration. As initial conclusions, we stress the fact that the inclusion of quantum effects in this CGHS-inspired cosmology is responsible for an additional transfer of energy from the inflaton to the matter fields, promoting a faster transition to a matter dominated 2D universe, registered also by the fact that $\ddot{a}$ assume lower values in that case. The model where $q = 0$ and $\gamma = 1$ [12] was also considered but the results are not sensibly different from the $(q = 2, \gamma = 6)$ version presented above.

IV - 2D cosmology with fermions . – In this section we investigate the effects produced by a combination of fermions and bosons in a 2D cosmology, including quantum corrections. In this case the fermionic field represents a matter constituent and the bosonic field play, as in the previous model, the role of the inflaton. The total action of the new theory is written as $S_T = S_{cl} + S_{qt} + S_F$,; where the bosonic classical action includes now only one source (the inflaton f). S_F is the (matter) fermionic action; it is given by

$$S_F = \frac{1}{2\pi}\int d^2x\sqrt{-g}\left\{\frac{i}{2}\left[\bar{\psi}\Gamma^\mu D_\mu\psi - (D_\mu\bar{\psi})\Gamma^\mu\psi\right] - V\right\} \tag{11}$$

where in the last term we consider an self-interacting fermionic potential $V(\psi,\bar{\psi})$. The gravitational equations of motion $G_{\mu\nu} = T_{\mu\nu}$ include now a fermionic term in the total energy-momentum tensor. From Noether theorem:

$$T^F_{\mu\nu} = -\frac{i}{4}\left[\bar{\psi}\Gamma_\mu D_\nu\psi + \bar{\psi}\Gamma_\nu D_\mu\psi - D_\nu\bar{\psi}\Gamma_\mu\psi - D_\mu\bar{\psi}\Gamma_\nu\psi\right] + g_{\mu\nu}\left\{\frac{i}{2}\left[\bar{\psi}\Gamma^\alpha D_\alpha\psi - D_\alpha\bar{\psi}\Gamma^\alpha\psi\right] - V\right\} \tag{12}$$

The usual Dirac matrices (γ^μ) become, due to the general covariance principle [7]:

$$\Gamma^0 = \gamma^0,\ \Gamma^i = \frac{1}{a(t)}\gamma^i,\ \Gamma^3 = -\imath\sqrt{-g}\,\Gamma^0\Gamma^1\Gamma^2 = \gamma^3. \tag{13}$$

Following a path analogous to the one in section III, we obtain two independent gravitational field equations, namely

$$\ddot{\phi} + \frac{\dot{a}}{a}\dot{\phi} = 2\dot{\phi}^2 - \frac{1}{4}e^{2\phi}\left[2V - \left(\bar{\psi}\frac{dV}{d\bar{\psi}} + \frac{dV}{d\psi}\psi\right)\right] - \frac{\kappa}{4}e^{2\phi}\left[\gamma\left(\frac{\dot{a}}{a}\dot{\phi} + \ddot{\phi}\right) - 2\frac{\ddot{a}}{a}\right], \tag{14}$$

$$\dot{\phi}^2 - \frac{\ddot{a}}{2a} = \frac{\kappa}{4}e^{2\phi}\left[(\gamma - q)\left(\frac{\dot{a}}{a}\dot{\phi} + \ddot{\phi}\right) + (\gamma - 4)\frac{\ddot{a}}{2a}\right] + \frac{1}{4}e^{2\phi}\left[2V - \left(\bar{\psi}\frac{dV}{d\bar{\psi}} + \frac{dV}{d\psi}\psi\right)\right]. \tag{15}$$

The dilaton and bosonic fields evolution is ruled again by expressions (8) and (9), with $\theta = 0$. For the fermionic field we have the curved space-time Dirac equations [7] $\dot{\psi} + \frac{1}{2}\frac{\dot{a}}{a}\psi + i\gamma^0\frac{dV}{d\bar{\psi}} = 0$, with an analogous expression for $\bar{\psi}$. Using the explicit form of the fermionic potential $V = (\bar{\psi}\gamma^3\psi)^{2n}$ and the bi-spinor components (ψ_A, ψ_B), we can write the field equations more coveniently as

$$\ddot{\phi} + \frac{\dot{a}}{a}\dot{\phi} = 2\dot{\phi}^2 - \frac{\kappa}{4}e^{2\phi}\left[\gamma\left(\frac{\dot{a}}{a}\dot{\phi} + \ddot{\phi}\right) - 2\frac{\ddot{a}}{a}\right] + \frac{1}{2}e^{2\phi}(2n-1)(i\psi_B^*\psi_A - i\psi_A^*\psi_B)^{2n}, \tag{16}$$

$$\dot{\phi}^2 - \frac{\ddot{a}}{2a} = \frac{\kappa}{4}e^{2\phi}\left[(\gamma - q)\left(\frac{\dot{a}}{a}\dot{\phi} + \ddot{\phi}\right) + (\gamma - 4)\frac{\ddot{a}}{2a}\right] - \frac{1}{2}e^{2\phi}(2n-1)(i\psi_B^*\psi_A - i\psi_A^*\psi_B)^{2n}, \tag{17}$$

$$\dot{\psi}_A + \frac{1}{2}\frac{\dot{a}}{a}\psi_A + 2n(i\psi_B^*\psi_A - i\psi_A^*\psi_B)^{2n-1}\psi_B = 0, \tag{18}$$

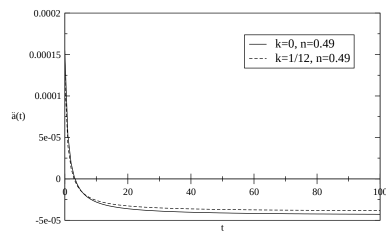
with an analogous expression for ψ_B. The initial conditions were chosen, again, in such a manner that the 2D universe starts dominated by an inflaton field with initial energy density $\rho_f(0) = 1$. The initial energy density of the fermionic field was fixed so that matter is created at expenses of the inflaton and gravitational fields energy (we do not consider a direct interaction between the sources in this case). The values adopted for these initial conditions were $a(0) = 1$, $\quad \dot{a}(0) = 1$, $\quad \phi(0) = 0$, $\quad \dot{\phi}(0) = -0.01, f(0) = 0$, $\quad \dot{f}(0) = 2.0$, $\quad \psi_A(0) = -0.1i, \psi_B(0) = 0.01$.

The values chosen for the quantum correction parameters coincide with those of the previous section $\kappa = 1/12$, $q = 2$ and $\gamma = 6$ [12]. We start the numerical analysis by describing the behavior of the 2D universe when the power (n) in the fermionic potential $V(\bar{\psi},\psi)$ was chosen to be $n = 1/2$ and $n = 1$. These values correspond to special cases of the Nambu-Jona-Lasinio potential [8]. What is verified in these cases is a behaviour completely analogous to the strictly bosonic model, showing no distinct features in the classical nor in the quantum corrected versions. An important point here is that there is no transition in the acceleration evolution, meaning that an entrance in a matter dominated/decelerated period (following inflation) is not possible in this case.

The most interesting result appears, however, when we choose the power n to be in the neigborhood of values $n \approx 0.49$. Starting with the behavior of the scale factor $a(t)$, making initially $\kappa = 0$, what we verify is that we have an analogous situation to the strictly bosonic case (see figure 1); we have a fast expansion and this depends strongly on the initial value of the time derivative of the dilaton field ϕ. When we turn on the quantum effects the expansion becomes slower, again, as in the previous model.

Next, we analyze the behavior of the fermion energy density ρ_ψ and the inflaton energy density ρ_f that are given by $\rho_f = \frac{1}{4}\dot{\chi}^2 - \frac{\kappa}{2}\left[\left(\frac{\dot{a}}{a}\right)^2 - \gamma\dot{\phi}\frac{\dot{a}}{a} + q\dot{\phi}^2\right]$, $\rho_\psi = (i(\psi_B^*\psi_A - \psi_A^*\psi_B))^{2n}$. The inflaton density behaves again as in figure c; the fundamental point here is that the fermionic matter density is evolving due to the energy transference from the inflaton (via gravitational field). On the other hand, it is decreasing in absolute values due to the quick expansion of the 2D universe (figure 1). The quantum effects promote an additional transference of energy from the inflaton to the matter field, like in the strictly bosonic case.

The behavior of the acceleration follows the bosonic case patterns when one focus on the dilaton field: increasing values of $\dot{\phi}(0)$ imply into a smoother decay of the acceleration. When we consider quantum effects there is a more drastic fall in the values of $\ddot{a}$. The most important feature of the case $n = 0.49$ is that we have a transition from the accelerated regime to a decelerated period (see figure e) showing that the inclusion of fermionic matter as a constituent permit a gradual exit form the inflation period. Besides, the quantum contributions decay as consequence of the 2D universe expansion, approa



(e) acceleration vs time comparing classical with quantum corrected solutions

Our final comments stress the fact that the inclusion of quantum effects in this 2D cosmology is responsible for an additional transfer of energy from the inflaton to the matter fields, promoting a faster transition to a matter dominated 2D universe, although always with positive acceleration in the bosonic case. In the fermionic case we verify that the inclusion of the curved space-time Dirac dynamics is responsible for an interesting transition to a *decelerated* regime, dominated by fermionic matter. Again,the model where $q = 0$ and $\gamma = 1$ was also considered but the results are not sensibly different from the $(q = 2, \gamma = 6)$ case.

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